International Civil Aviation Organization

The Fifth Meeting of ICAO Asia/Pacific Air Traffic Flow Management Steering Group (ATFM/SG/5)

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Agenda Item 4: Review of Current CDM/ATFM Operations and Problem Areas

PRELIMINARY DELAY ASSESSMENT BASED ON SIMPLE MATHEMATICAL MODEL

(Presented by Hong Kong China)

SUMMARY

This paper presents a methodology, by application of simple mathematical model, to preliminarily assess the delay caused by demand/capacity imbalance at an airport. The model is simple and generic in nature, it is not expected to provide an accurate delay estimation figure at a specific airport tactically or pre-tactically. But it may act as a reference for States to preliminarily assess the necessity and urgency of establishing appropriate ATFM mechanism for their airports.

1. INTRODUCTION

1.1 Despite States continuously monitor the traffic demand growth and the occurrence of runway capacity constraint events at airports, it is not obvious how to project and quantify the implication.

1.2 A comprehensive study of a specific airport could provide a sound conclusion, but it may also involve considerable amount of resource and expertise. To address this issue, a simple mathematical model is introduced for reference.

1.3 The model is designed to translate runway capacity and duration of constraint into average delay for arrival traffic. As the model is simple and generic in nature, it is not expected to provide an accurate delay estimation figure at a specific airport tactically or pre-tactically. But it may act as a reference for States to preliminarily assess the necessity and urgency of establishing appropriate ATFM mechanisms for airports.

2. DISCUSSION

<u>Components for Delay</u>

2.1 There are many components which may contribute to the delay of an arrival flight, e.g., multiple arrival flights are scheduled for the same slot time, schedule slippage and demand/capacity imbalance. The first 2 components are beyond the scope of this IP, only the delay generated by demand/capacity imbalance will be discussed in this paper.

Mathematical Model

2.2 The model is based on the following assumptions:

- The time intervals between arrival schedule slots are equal to 60/Demand minutes, where Demand is the number of scheduled arrival flights in an hour;
- The ETA of arrival flights are perfectly adhered to the schedule.
- The time intervals between the actual landing time of flights are equal to 60/Capacity, where Capacity is the number of flights that can land under prevalent condition within an hour.
- The delay for flights scheduled to arrive during the recovery period is not considered in the model.

Outcome

2.3 When Capacity is no greater than Demand, the average delay for the flights which scheduled to arrive the airport within the period of capacity constraint is found to be:

Avg Delay(min)= $30 * \mathrm{h} * (\frac{\mathrm{D}}{\mathrm{C}} - 1)$

Where h is the duration of capacity constraint, D is the demand and C is runway capacity.

2.4 The maximum delay for the same group of flights is found to be:

Max Delay(min)= 2*Avg Delay

2.5 The maximum number of flights in holding pattern in the model is found to be:

Max Holding = Model Max Holding Factor * D

Where Model Max Holding Factor= $\mathrm{h}(1 - \frac{\mathrm{C}}{\mathrm{D}})$

2.6 The formulae are evaluated and tabulated as follows (**Tables 1, 2 and 3**):

Runway Capacity (% of Demand)	Average Delay (min)	Max Delay (min)	Model Max Holding Factor
100%+	0	0	0
95%	2	3	0.05
90%	3	7	0.1
85%	5	11	0.15
80%	8	15	0.2
75%	10	20	0.25
70%	13	26	0.3
65%	16	32	0.35
60%	20	40	0.4
55%	25	49	0.45

50%	30	60	0.5

Table 1: Duration of Capacity Constraint = 1 hour

Runway Capacity (% of Demand)	Average Delay (min)	Max Delay (min)	Model Max Holding Factor
100%+	0	0	0
95%	3	6	0.1
90%	7	13	0.2
85%	11	21	0.3
80%	15	30	0.4
75%	20	40	0.5
70%	26	51	0.6
65%	32	65	0.7
60%	40	80	0.8
55%	49	98	0.9
50%	60	120	1

Table 2: Duration of Capacity Constraint = 2 hour

Runway Capacity (% of Demand)	Average Delay (min)	Max Delay (min)	Model Max Holding Factor
100%+	0	0	0
95%	5	9	0.15
90%	10	20	0.3
85%	16	32	0.45
80%	23	45	0.6
75%	30	60	0.75
70%	39	77	0.9
65%	48	97	1.05
60%	60	120	1.2
55%	74	147	1.35
50%	90	180	1.5

Table 3: Duration of Capacity Constraint = 3 hour

2.7 The average delay, max delay and the maximum number of flights in holding pattern derived from the tables are referring to the flights in the model. In real life, the actual figures particularly the maximum number of flights in holding pattern may differ from the theoretical values significantly.

2.8 The details of the mathematical deduction are stipulated in **Attachment A**.

An Example of Application

2.9 An airport possesses dual runways, the maximum runway capacity for arrival is 40 flights per hour. The current arrival traffic demand is 28 flights per hour. Typically, the airport encounters 30 days per annual with bad weather, which reduce the runway capacity by 20% for 2 hours. About 10 days per year, it encounters poor weather which reduces the runway capacity by 30% for 3 hours. The traffic growth of the airport is estimated to be 8%p.a. for the next few years.

2.10 If the math model is applied, the relevant figures are tabulated as follows (**Table 4**):

			Bad Days(30 days p.a.)				Poor Days(10 days p.a.)			
	Max Capacity	Dmd	Cap	Avg Delay	Max Delay	Holding Demand	Cap	Avg Delay	Max Delay	Holding Demand
Y0	40	28	32	0	0	0	28	0	0	0
Y1	40	30	32	0	0	0	28	7	14	7
Y2	40	33	32	1	2	1	28	15	30	14
Y3	40	35	32	6	12	7	28	23	47	22

Table 4: Projected Delays in Bad Weather – 8% p.a. Traffic Growth

2.11 In the current year, there is no significant delay induced by both bad and poor weather days. In the next year Y1, the airport will encounter 10 days with slight delay to arrival flights. However the situation gets worse quickly in the subsequent years.

2.12 When the traffic demand is very much well below the maximum runway capacity, the air traffic may still operate smoothly even in poor weathers for years with robust traffic growth. There may not be any indication of a capacity problem. Once a threshold is passed, the situation could deteriorate quickly. The threshold in this example is when the traffic demand is over the poor day runway capacity.

3. ACTION BY THE MEETING

The meeting is invited to:

a) note the information contained in this paper; and

b) discuss any relevant matters as appropriate.

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ATTACHMENT A: Deduction of Formulae

Let D be the arrival traffic demand of an airport, i.e. the number of arrival flights scheduled in an hour;

Let C be the runway capacity an airport for arrival, i.e. the number of flights landed in an hour;

If C> D then obviously there is no delay for any flight

If C<=D
Let S_i be the scheduled arrival time of the i^{th} flight
Let A_i be the actual landing time of the i^{th} flight

Assume the schedule slots are spread evenly across the time line, the time interval between 2 schedule slots is 60/D minutes;

Assume the actual landing times are spread evenly across the time line, the time interval between the actual landing times of 2 flights is 60/C minutes

Average Delay

Let $S_1=A_1=0$;
$S_2=60/D$
…
$S_i=(i-1)*60/D$

Similarly, $A_i=(i-1)*60/C$
The delay experienced by the i^{th} flight is $A_i-S_i=(i-1)*60*(\frac{1}{C}-\frac{1}{D})$

Let n be the total number of flights scheduled to arrive within the period of capacity constraint,

Total Delay $=\sum_{i=1}^{n}(Ai-Si)=\sum_{i=1}^{n}(i-1)*60*(\frac{1}{C}-\frac{1}{D})$

$=60*(\frac{1}{C}-\frac{1}{D})\sum_{i=1}^{n}(i-1)$

$=60*\left(\frac{1}{C}-\frac{1}{D}\right)*n*(n-1)/2$

Avg Delay =Total Delay/n

$=30*\left(\frac{1}{C}-\frac{1}{D}\right)*(n-1)$

Since n=h*D where h is the duration of capacity constraint in terms of hour;

$=30*\left(\frac{1}{C}-\frac{1}{D}\right)*(hD-1)$

$\approx 30*\left(\frac{1}{C}-\frac{1}{D}\right)*hD$

$=30*h*(\frac{D}{C}-1)$

Maximum Delay

Since the delay experienced by the i^{th} flight is A_i-S_i $=(i-1)*60*(\frac{1}{C}-\frac{1}{D})$

The last flight, i.e. the n^{th} flight, will be subject to the maximum delay,

Max Delay $=(n-1)*60*(\frac{1}{C}-\frac{1}{D})$
$=2*30*(n-1)*(\frac{1}{C}-\frac{1}{D})$
=2*Avg. Delay

Maximum Number of Holding Flights

Let r be the elapsed time from the holding fix to the runway in terms of minutes;

A flight will enter the holding pattern at time $S_i - r$
The flight will leave the holding pattern at time $A_i - r$

At any specific time t , a flight will be inside the holding pattern if and only if

The flight has entered but not yet left the hold and delay is 4 minutes or more
i.e.

$S_i - r \leq t \leq A_i - r$ and A_i-$S_i \geq 4$

A_i-$S_i \geq 4$

$$(i-1)*60*\left(\frac{1}{C}-\frac{1}{D}\right) \geq 4$$
$$(i-1)*\frac{2}{hD}*30*h*\left(\frac{D}{C}-1\right) \geq 4$$
$$(i-1)*\frac{2}{hD}*\text{Avg Delay} \geq 4$$
$$i \geq \frac{2hD}{\text{Avg. Delay}}+1$$

AND

$S_i - r \leq t$

$$(i-1)*\frac{60}{D}-r \leq t$$

$$i \leq \frac{(t+r)D}{60}+1$$

AND

$t \leq A_i - r$

$t \leq (i-1) * \frac{60}{C} - r$

$i \geq \frac{(t+r)C}{60} + 1$

The necessary and sufficient condition for the i^{th} flight is inside the holding pattern at time t is

$$\frac{(t+r)D}{60} + 1 \geq i \geq MAX\left(\frac{(t+r)C}{60} + 1, \frac{2hD}{\text{Avg. Delay}} + 1\right)$$

The maximum number of flights in holding pattern occurs when the last flight enters the hold, i.e. when $t = S_n - r = (n-1) * \frac{60}{D} - r$

$$\frac{(t+r)D}{60} + 1 \geq i \geq MAX\left(\frac{(t+r)C}{60} + 1, \frac{2hD}{\text{Avg. Delay}} + 1\right) \geq \frac{(t+r)C}{60} + 1$$

Substitute t with S_n-r, we get

$$n \geq i \geq (n-1) * \frac{C}{D} + 1$$

The maximum number of flights inside holding pattern cannot exceed

$$n - \left[(n-1) * \frac{C}{D} + 1\right]$$

$$= (n-1)\left(1 - \frac{C}{D}\right)$$

$$\approx n\left(1 - \frac{C}{D}\right)$$

$$= h\left(1 - \frac{C}{D}\right) * D$$

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